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International Baccalaureate®
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MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 1

Candidate session number

0	0							

Thursday 9 May 2013 (afternoon)

1 hour 30 minutes

Examination code

2	2	1	3	–	7	4	0	5

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematical Studies SL*** *information booklet* is required for this paper.
- Answer all questions.
- Write your answers in the boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is *[90 marks]*.

16 pages

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. A cuboid has the following dimensions: length $= 8.7$ cm, width $= 5.6$ cm and height $= 3.4$ cm.

(a) Calculate the **exact** value of the volume of the cuboid, in cm^3. *[2 marks]*

(b) Write your answer to part (a) correct to

(i) one decimal place;

(ii) three significant figures. *[2 marks]*

(c) Write your answer to **part (b)(ii)** in the form $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$. *[2 marks]*

Working:

Answers:

(a) . . . . . .

(b) (i) . . . . . .

(ii) . . . . . .

(c) . . . . . .

2. Consider the following propositions.

p : *Students stay up late.*
q : *Students fall asleep in class.*

(a) Write the following compound proposition in symbolic form. *[2 marks]*

If students do not stay up late then they will not fall asleep in class.

(b) Complete the following truth table.

p	q	$\neg q$	$p \vee \neg q$	$\neg(p \vee \neg q)$
T	T			
T	F			
F	T			
F	F			

[3 marks]

(c) Write down a reason why the statement $\neg(p \vee \neg q)$ is not a contradiction. *[1 mark]*

Working:

Answers:

(a) . . . . . .

(c) . . . . . .

3. Consider the numbers 3, -5, $\sqrt{7}$, 2^{-3} and 1.75.
Complete the table below, placing a tick (✓) to show which of the number sets, $\mathbb{N}$, $\mathbb{Q}$ and $\mathbb{R}$ these numbers belong to. The first row has been completed as an example.

	$\mathbb{N}$	$\mathbb{Q}$	$\mathbb{R}$
3	✓	✓	✓
-5			
$\sqrt{7}$			
2^{-3}			
1.75			

[6 marks]

Working:

4. The table shows the number of bicycles owned by 50 households.

Number of bicycles per household	Frequency (number of households)	Cumulative frequency
0	3	3
1	7	10
2	12	22
3	14	36
4	4	40
5	t	w
6	2	50

(a) Write down the value of

(i) t;

(ii) w. *[2 marks]*

(b) Indicate with a tick (✓) whether the following statements are True or False.

Statement	True	False
Every household owns at least 1 bicycle.		
The median number of bicycles per household is 3.		
The 25th percentile is 1 bicycle per household.		
There are 10 households with at most 1 bicycle.		

[4 marks]

Working:

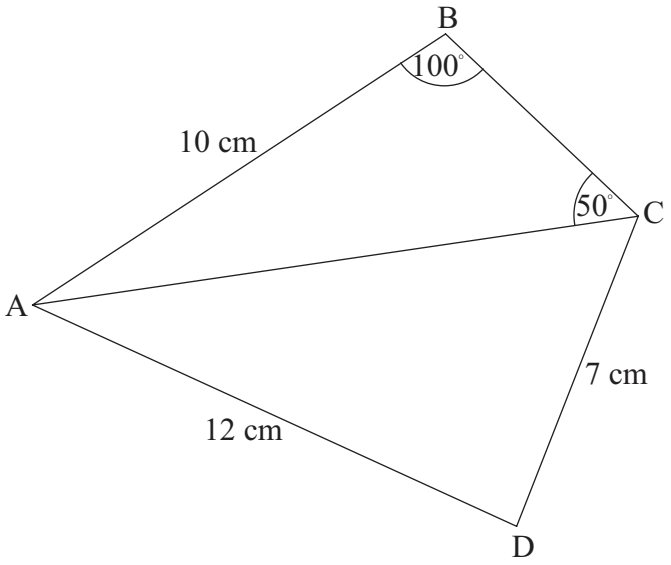
Answers:

(a) (i) . . . . . .

(ii) . . . . . .

5. The quadrilateral ABCD has $AB = 10\text{ cm}$, $AD = 12\text{ cm}$ and $CD = 7\text{ cm}$.
The size of angle ABC is 100° and the size of angle ACB is 50°.

diagram not to scale



(a) Find the length of AC in centimetres. *[3 marks]*

(b) Find the size of angle ADC. *[3 marks]*

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

6. A market researcher surveyed men and women about their preferred holiday destination. The holiday destinations were Antigua, Barbados, Cuba, Guadeloupe and Jamaica. A χ^2 test for independence was conducted at the 5 % significance level. The χ^2 calculated value was found to be 8.73.

(a) Write down the null hypothesis. *[1 mark]*

(b) Find the number of degrees of freedom for this test. *[2 marks]*

(c) Write down the critical value for this test. *[1 mark]*

(d) State the conclusion of this test. Give a reason for your decision. *[2 marks]*

Working:

Answers:

(a) . . . . . .

. . . . . .

(b) . . . . . .

(c) . . . . . .

(d) . . . . . .

. . . . . .

7. Consider the function $f(x) = -2\cos(x) + 1$ where $-180^\circ \le x \le 360^\circ$.

(a) For the function $f(x)$, write down the

(i) period;

(ii) amplitude. *[2 marks]*

(b) Find the range of $f(x)$. *[2 marks]*

(c) Find the number of solutions of the equation $f(x) = 1$, in the given domain. *[2 marks]*

Working:

Answers:

(a) (i) . . . . . .

(ii) . . . . . .

(b) . . . . . .

(c) . . . . . .

8. The equation of a line L_1 is $2x + 5y = -4$.

(a) Write down the gradient of the line L_1. *[1 mark]*

A second line L_2 is perpendicular to L_1.

(b) Write down the gradient of L_2. *[1 mark]*

The point $(5, 3)$ is on L_2.

(c) Determine the equation of L_2. *[2 marks]*

Lines L_1 and L_2 intersect at point P.

(d) Using your graphic display calculator or otherwise, find the coordinates of P. *[2 marks]*

Working:

Answers:

(a) . . . . . .

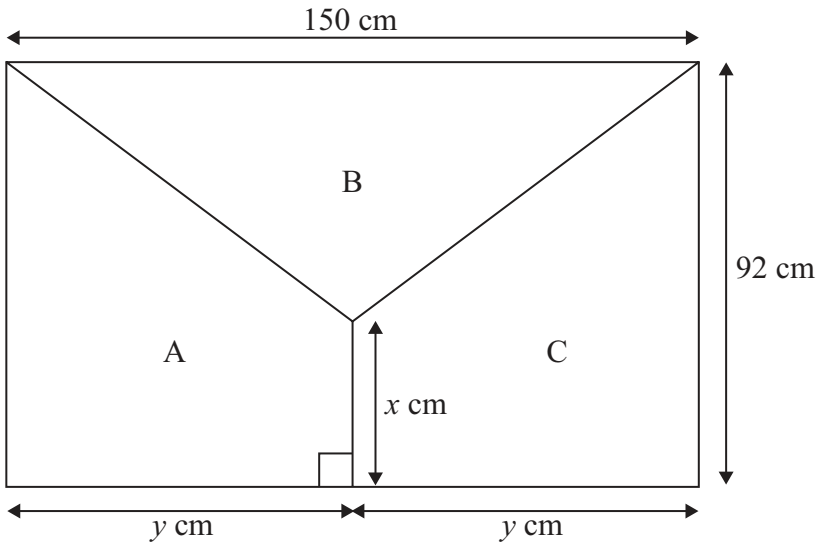
(b) . . . . . .

(c) . . . . . .

(d) . . . . . .

9. The diagram below represents a rectangular flag with dimensions 150 cm by 92 cm. The flag is divided into three regions A, B and C.

diagram not to scale



(a) Write down the total area of the flag. *[1 mark]*

(b) Write down the value of y. *[1 mark]*

The areas of regions A, B, and C are equal.

(c) Write down the area of region A. *[1 mark]*

(d) Using your answers to **parts (b) and (c)**, find the value of x. *[3 marks]*

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

(d) . . . . . .

10. Alan's laundry basket contains two green, three red and seven black socks.
He selects one sock from the laundry basket at random.

(a) Write down the probability that the sock is red. *[1 mark]*

Alan returns the sock to the laundry basket and selects two socks at random.

(b) Find the probability that the first sock he selects is green and the second sock is black. *[2 marks]*

Alan returns the socks to the laundry basket and again selects two socks at random.

(c) Find the probability that he selects two socks of the same colour. *[3 marks]*

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

11. A curve is described by the function $f(x) = 3x - \frac{2}{x^2}$, $x \neq 0$.

(a) Find $f'(x)$. *[3 marks]*

The gradient of the curve at point A is 35.

(b) Find the x-coordinate of point A. *[3 marks]*

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

12. Yoshi is spending a year travelling from Japan to Italy and then to the United States of America.
Before Yoshi leaves Japan he changes 100 000 Japanese Yen (JPY) into euro (EUR).
The exchange rate is 1 JPY = 0.006 EUR.

(a) Calculate the amount Yoshi receives, in EUR. *[2 marks]*

Yoshi spends 426.70 EUR in Italy. In an American bank he changes the remaining amount, into US dollars (USD), at an exchange rate of 1 USD = 0.673 EUR.
The bank charges 1.5 % commission.

(b) Calculate the amount, in USD, Yoshi receives after commission.
Give your answer correct to the nearest USD. *[4 marks]*

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

13. The number of bacteria in a colony is modelled by the function

$$N(t) = 800 \times 3^{0.5t},\ t \geq 0,$$

where N is the number of bacteria and t is the time in hours.

(a) Write down the number of bacteria in the colony at time $t = 0$. *[1 mark]*

(b) Calculate the number of bacteria present at 2 hours and 30 minutes. Give your answer correct to the nearest hundred bacteria. *[3 marks]*

(c) Calculate the time, in hours, for the number of bacteria to reach 5500. *[2 marks]*

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

14. The number of passengers in the first ten carriages of a train is listed below.

$$6, 8, 6, 3, 8, 4, 8, 5, p, p$$

The mean number of passengers per carriage is 5.6.

(a) Calculate the value of p. *[2 marks]*

(b) Find the median number of passengers per carriage. *[2 marks]*

If the passengers in the eleventh carriage are also included, the mean number of passengers per carriage increases to 6.0.

(c) Determine the number of passengers in the eleventh carriage of the train. *[2 marks]*

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

15. Marcus has been given 500 Australian dollars (AUD) by his grandmother for his 18th birthday.
He plans to deposit it in a bank which offers a nominal annual interest rate of 6.0 %, **compounded quarterly**, for three years.

(a) Calculate the total amount of interest Marcus would earn, in AUD, over the three years. **Give your answer correct to two decimal places**. *[3 marks]*

Marcus would earn the same amount of interest, **compounded annually**, for three years if he deposits the 500 AUD in a second bank.

(b) Calculate the interest rate the second bank offers. *[3 marks]*

Working:

Answers:

(a) . . . . . .

(b) . . . . . .